Integration- Questions

June 2018 Mathematics Advanced Paper 1: Pure Mathematics 1

1.

Given that $k \in \mathbb{Z}^+$

(a) show that $\int_k^{3k} \frac{2}{(3x-k)} \mathrm{d}x$ is independent of k,

(4)

(b) show that $\int_k^{2k} \frac{2}{(2x-k)^2} \mathrm{d}x$ is inversely proportional to k.

(3)

2.

The height above ground, H metres, of a passenger on a roller coaster can be modelled by the differential equation

$$\frac{\mathrm{d}H}{\mathrm{d}t} = \frac{H\cos(0.25t)}{40}$$

where t is the time, in seconds, from the start of the ride.

Given that the passenger is 5 m above the ground at the start of the ride,

(a) show that $H = 5\mathrm{e}^{0.1\sin(0.25t)}$

(5)

(b) State the maximum height of the passenger above the ground.

(1)

The passenger reaches the maximum height, for the second time, T seconds after the start of the ride.

(c) Find the value of T.

(2)

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3.

3. (*a*) $y = 5^x + \log_2(x + 1), \quad 0 \leqslant x \leqslant 2$

Complete the table below, by giving the value of y when $x = 1$

x	0	0.5	1	1.5	2
y	1	2.821		12.502	26.585

(1)

(*b*) Use the trapezium rule, with all the values of y from the completed table, to find an approximate value for

$$\int_0^2 (5^x + \log_2(x+1))\,dx$$

giving your answer to 2 decimal places.

(4)

(*c*) Use your answer to part (*b*) to find an approximate value for

$$\int_0^2 (5 + 5^x + \log_2(x+1))\,dx$$

giving your answer to 2 decimal places.

(1)

4.

10.

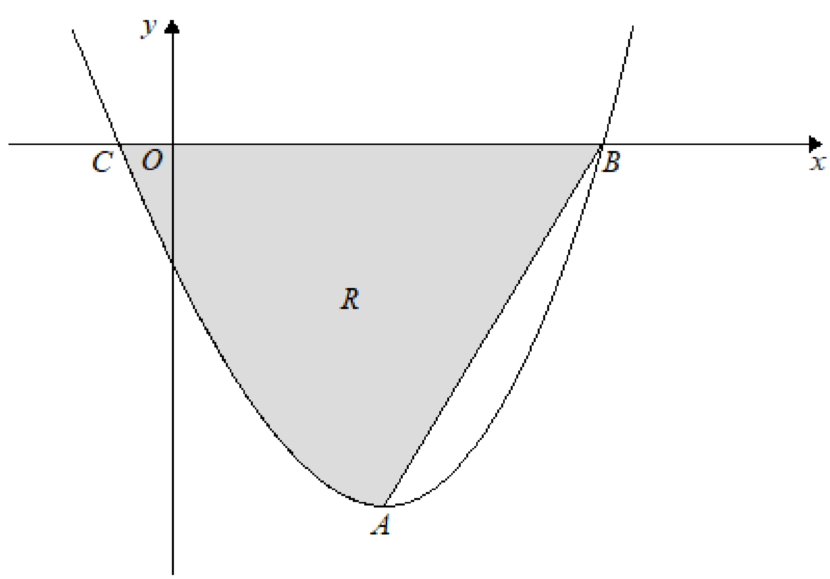

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = 4x^3 + 9x^2 - 30x - 8, \qquad -0.5 \leqslant x \leqslant 2.2$$

The curve has a turning point at the point A.

(a) Using calculus, show that the x coordinate of A is 1

(3)

The curve crosses the x-axis at the points B (2, 0) and C $\left(-\frac{1}{4}, 0\right)$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the line AB, and the x-axis.

(b) Use integration to find the area of the finite region R, giving your answer to 2 decimal places.

(7)

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5.

2. The curve C has equation

$$y = 8 - 2^{x-1}, \qquad 0 \leq x \leq 4.$$

(a) Complete the table below with the value of y corresponding to $x = 1$

x	0	1	2	3	4
y	7.5		6	4	0

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to find an approximate value for $\int_0^4 (8 - 2^{x-1})\ \mathrm{d}x$.

(3)

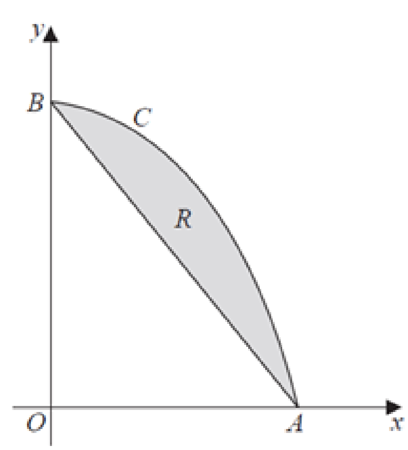

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = 8 - 2^{x-1}, \quad 0 \leq x \leq 4$.

The curve C meets the x-axis at the point A and meets the y-axis at the point B.

The region R, shown shaded in Figure 1, is bounded by the curve C and the straight line through A and B.

(c) Use your answer to part (b) to find an approximate value for the area of R.

(2)

6.

7.

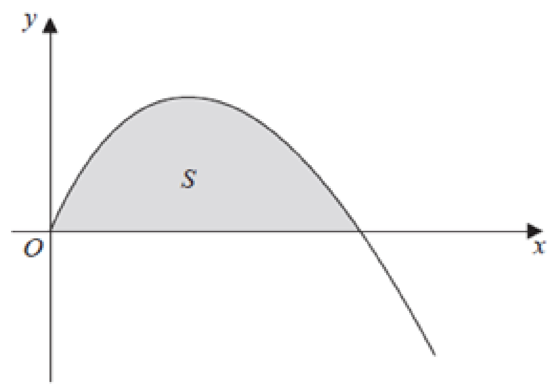

Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 3x - x^{\frac{3}{2}} \quad x \geq 0 .$$

The finite region S, bounded by the x-axis and the curve, is shown shaded in Figure 3.

(a) Find

$$\int \left(3x - x^{\frac{3}{2}} \right) \mathrm{d}x .$$

(3)

(b) Hence find the area of S.

(3)

7.

6. (*a*) Find

$$\int 10x(x^{\frac{1}{2}} - 2)\ \mathrm{d}x\,,$$

giving each term in its simplest form.

(4)

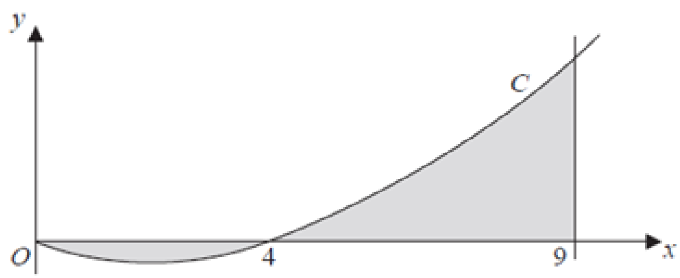

Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = 10x(x^{\frac{1}{2}} - 2), \qquad x \geq 0.$$

The curve C starts at the origin and crosses the x-axis at the point $(4, 0)$.

The area, shown shaded in Figure 2, consists of two finite regions and is bounded by the curve C, the x-axis and the line $x = 9$.

(*b*) Use your answer from part (*a*) to find the total area of the shaded regions.

(5)

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8.

1.

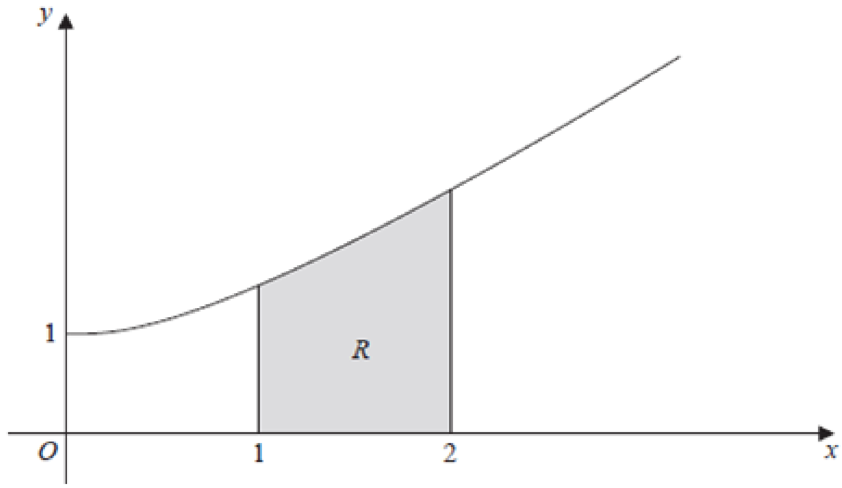

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \sqrt{(x^2 + 1)}$, $x \geq 0$.

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$.

The table below shows corresponding values for x and y for $y = \sqrt{(x^2 + 1)}$.

x	1	1.25	1.5	1.75	2
y	1.414		1.803	2.016	2.236

(*a*) Complete the table above, giving the missing value of y to 3 decimal places.

(1)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to find an approximate value for the area of R, giving your answer to 2 decimal places.

(4)

9.

4. Use integration to find

$$\int_1^{\sqrt{3}} \left(\frac{x^3}{6} + \frac{1}{3x^2} \right) dx,$$

giving your answer in the form $a + b\sqrt{3}$, where a and b are constants to be determined.

(5)

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10.

4.

$$y = \frac{5}{(x^2+1)}.$$

(*a*) Copy and complete the table below, giving the missing value of y to 3 decimal places.

x	0	0.5	1	1.5	2	2.5	3
y	5	4	2.5		1	0.690	0.5

(1)

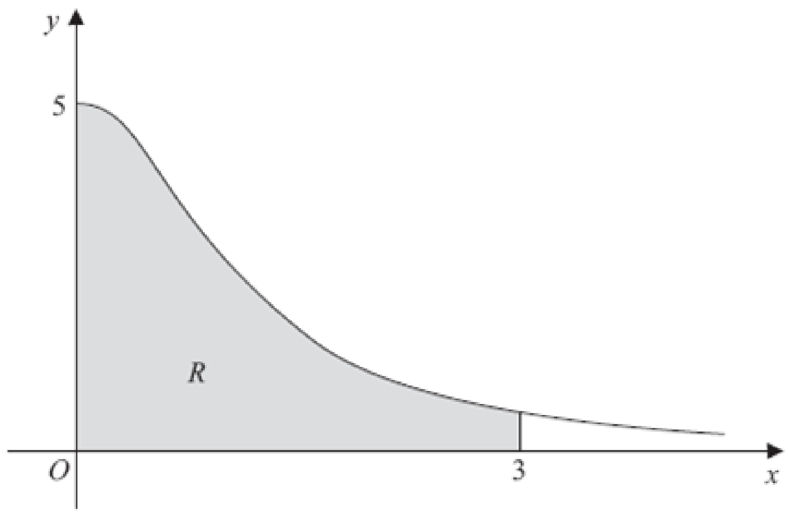

Figure 1

Figure 1 shows the region R which is bounded by the curve with equation $y = \frac{5}{(x^2+1)}$, the x-axis and the lines $x = 0$ and $x = 3$.

(*b*) Use the trapezium rule, with all the values of y from your table, to find an approximate value for the area of R.

(4)

(*c*) Use your answer to part (*b*) to find an approximate value for

$$\int_0^3 4 + \frac{5}{(x^2+1)}\, dx,$$

giving your answer to 2 decimal places.

(2)

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11.

9.

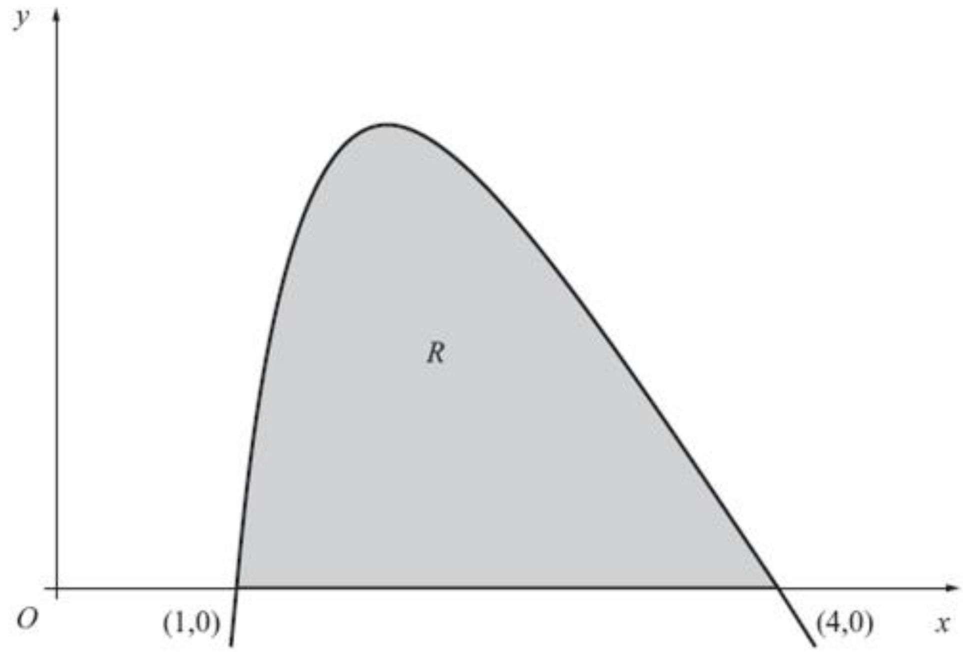

Figure 2

The finite region R, as shown in Figure 2, is bounded by the x-axis and the curve with equation

$$y = 27 - 2x - 9\sqrt{x} - \frac{16}{x^2}, \qquad x > 0.$$

The curve crosses the x-axis at the points (1, 0) and (4, 0).

(a) Copy and complete the table below, by giving your values of y to 3 decimal places.

x	1	1.5	2	2.5	3	3.5	4
y	0	5.866		5.210		1.856	0

(2)

(b) Use the trapezium rule with all the values in the completed table to find an approximate value for the area of R, giving your answer to 2 decimal places.

(4)

(c) Use integration to find the exact value for the area of R.

(6)

May 2012 Mathematics Advanced Paper 1: Pure Mathematics 2

12.

7.

$$y = \sqrt{(3^x + x)}$$

(*a*) Complete the table below, giving the values of y to 3 decimal places.

x	0	0.25	0.5	0.75	1
y	1	1.251			2

(2)

(*b*) Use the trapezium rule with all the values of y from your table to find an approximation for the value of

$$\int_0^1 \sqrt{(3^x + x)}\ \mathrm{d}x.$$

You must show clearly how you obtained your answer.

(4)

Jan 2012 Mathematics Advanced Paper 1: Pure Mathematics 2

13.

6.

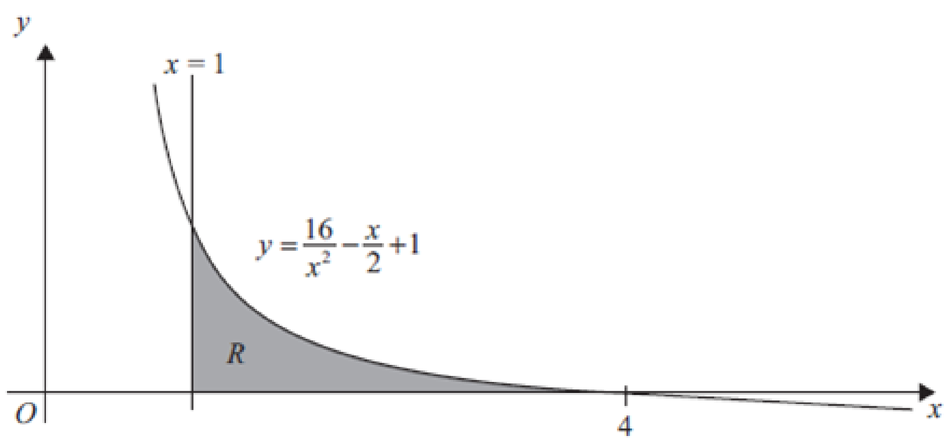

Figure 1

Figure 1 shows the graph of the curve with equation

$$y = \frac{16}{x^2} - \frac{x}{2} + 1, \qquad x > 0.$$

The finite region R, bounded by the lines $x = 1$, the x-axis and the curve, is shown shaded in Figure 1. The curve crosses the x-axis at the point (4, 0).

(*a*) Complete the table with the values of y corresponding to $x = 2$ and 2.5.

x	1	1.5	2	2.5	3	3.5	4
y	16.5	7.361			1.278	0.556	0

(2)

(*b*) Use the trapezium rule with all the values in the completed table to find an approximate value for the area of R, giving your answer to 2 decimal places.

(4)

(*c*) Use integration to find the exact value for the area of R.

(5)

Jan 2011 Mathematics Advanced Paper 1: Pure Mathematics 2

14.

6.

$$y = \frac{5}{3x^2 - 2}$$

(*a*) Copy and complete the table below, giving the values of y to 2 decimal places.

x	2	2.25	2.5	2.75	3
y	0.5	0.38			0.2

(2)

(*b*) Use the trapezium rule, with all the values of y from your table, to find an approximate value for $\int_2^3 \frac{5}{3x^2 - 2}\,dx$.

(4)

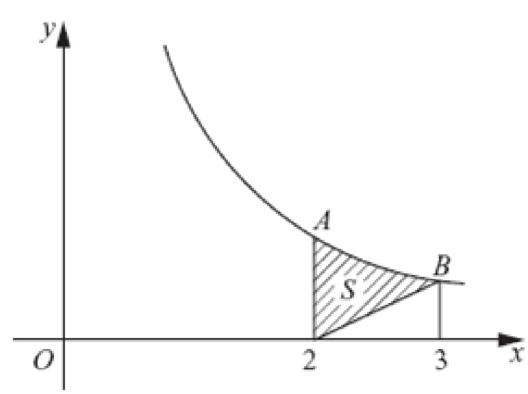

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \frac{5}{3x^2 - 2}$, $x > 1$.

At the points A and B on the curve, $x = 2$ and $x = 3$ respectively.

The region S is bounded by the curve, the straight line through B and (2, 0), and the line through A parallel to the y-axis. The region S is shown shaded in Figure 2.

(*c*) Use your answer to part (*b*) to find an approximate value for the area of S.

(3)

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15.

1. $y = 3^x + 2x.$

(*a*) Complete the table below, giving the values of y to 2 decimal places.

x	0	0.2	0.4	0.6	0.8	1
y	1	1.65				5

(2)

(*b*) Use the trapezium rule, with all the values of y from your table, to find an approximate value for $\int_0^1 (3^x + 2x)\ \mathrm{d}x$.

(4)

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16.

3.

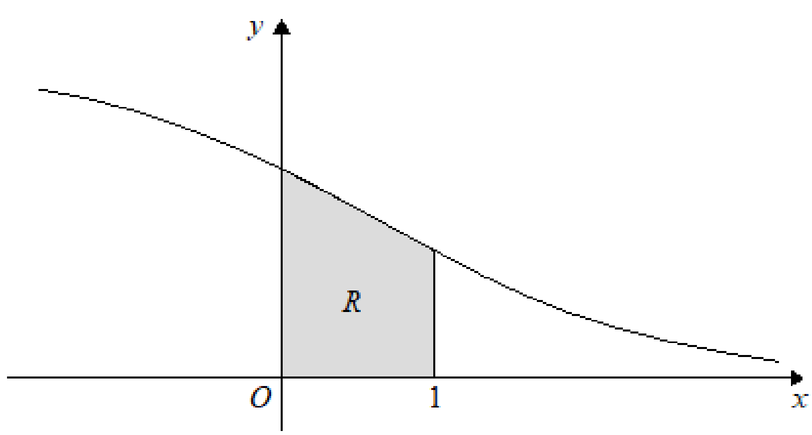

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \frac{6}{(e^x + 2)}$, $x \in \mathbb{R}$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the y-axis, the x-axis and the line with equation $x = 1$

The table below shows corresponding values of x and y for $y = \dfrac{6}{(e^x + 2)}$

x	0	0.2	0.4	0.6	0.8	1
y	2		1.71830	1.56981	1.41994	1.27165

(*a*) Complete the table above by giving the missing value of y to 5 decimal places.

(1)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to find an estimate for the area of R, giving your answer to 4 decimal places.

(3)

(*c*) Use the substitution $u = e^x$ to show that the area of R can be given by

$$\int_a^b \frac{6}{u(u+2)}\,du$$

where a and b are constants to be determined.

(2)

(*d*) Hence use calculus to find the exact area of R.
[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(6)

17.

5.

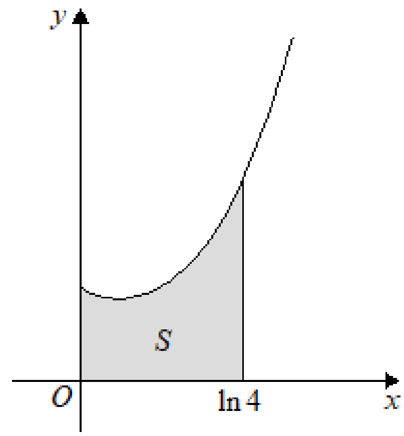

Diagram not drawn to scale

Figure 2

The finite region S, shown shaded in Figure 2, is bounded by the y-axis, the x-axis, the line with equation $x = \ln 4$ and the curve with equation

$$y = e^x + 2e^{-x}, \quad x \geqslant 0$$

The region S is rotated through 2π radians about the x-axis.

Use integration to find the exact value of the volume of the solid generated. Give your answer in its simplest form.

[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(7)

18.

7.

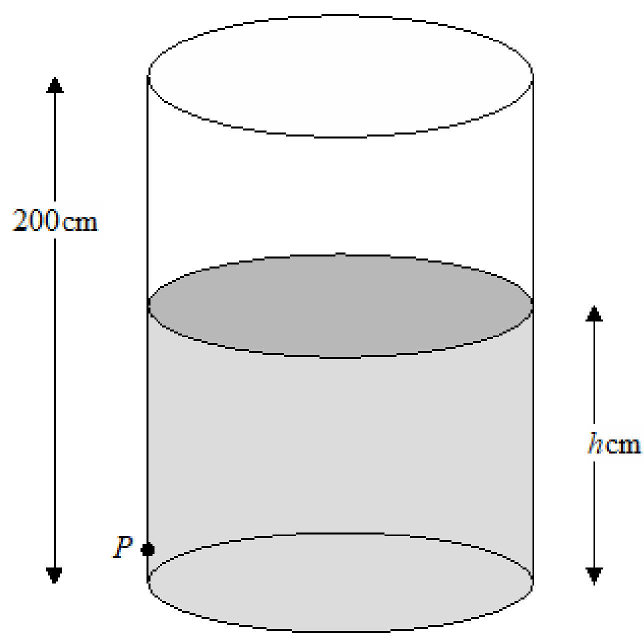

Diagram not drawn to scale

Figure 3

Figure 3 shows a vertical cylindrical tank of height 200 cm containing water.
Water is leaking from a hole P on the side of the tank.

At time t minutes after the leaking starts, the height of water in the tank is h cm.

The height h cm of the water in the tank satisfies the differential equation

$$\frac{dh}{dt} = k(h-9)^{\frac{1}{2}}, \qquad 9 < h \leqslant 200$$

where k is a constant.

Given that, when $h = 130$, the height of the water is falling at a rate of 1.1 cm per minute,

(a) find the value of k.

(2)

Given that the tank was full of water when the leaking started,

(b) solve the differential equation with your value of k, to find the value of t when $h = 50$

(6)

19.

8.

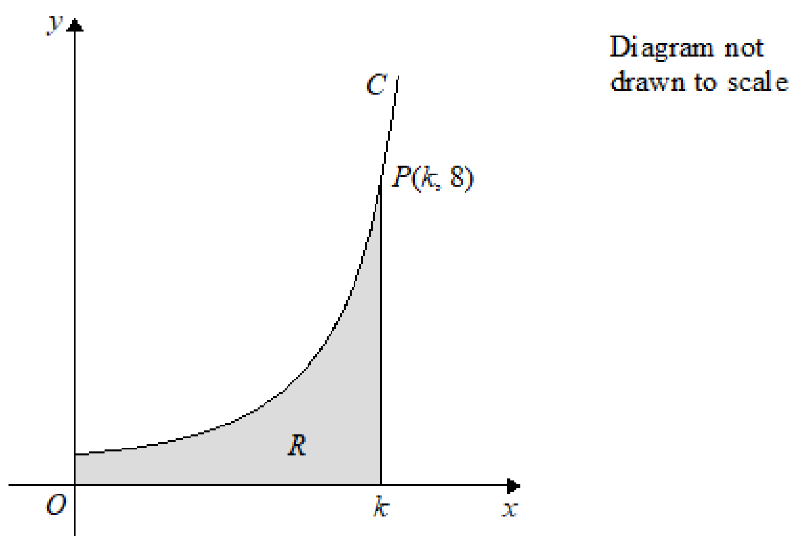

Figure 4

Figure 4 shows a sketch of part of the curve C with parametric equations

$$x = 3\theta\sin\theta, \qquad y = \sec^3\theta, \qquad 0 \leqslant \theta < \frac{\pi}{2}$$

The point $P(k, 8)$ lies on C, where k is a constant.

(a) Find the exact value of k.

(2)

The finite region R, shown shaded in Figure 4, is bounded by the curve C, the y-axis, the x-axis and the line with equation $x = k$.

(b) Show that the area of R can be expressed in the form

$$\lambda \int_{\alpha}^{\beta} (\theta \sec^2 \theta + \tan \theta \sec^2 \theta) \, d\theta$$

where λ, α and β are constants to be determined.

(4)

(c) Hence use integration to find the exact value of the area of R.

(6)

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20.

2.

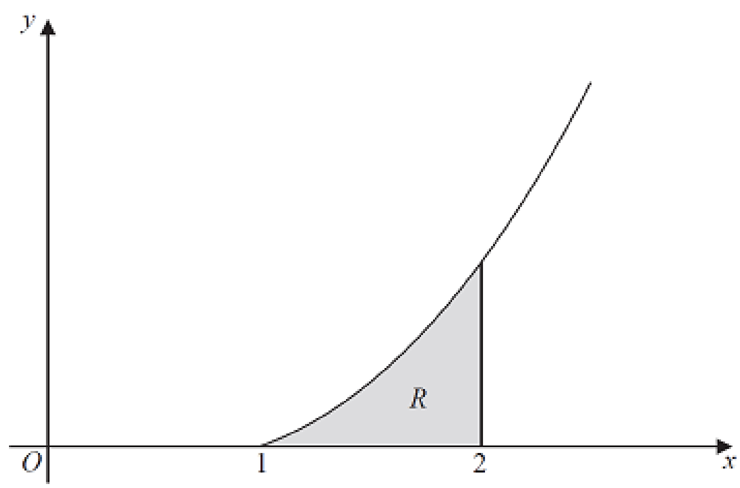

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = x^2 \ln x$, $x \geq 1$.

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 2$.

The table below shows corresponding values of x and y for $y = x^2 \ln x$.

x	1	1.2	1.4	1.6	1.8	2
y	0	0.2625		1.2032	1.9044	2.7726

(*a*) Complete the table above, giving the missing value of y to 4 decimal places.

(1)

(*b*) Use the trapezium rule with all the values of y in the completed table to obtain an estimate for the area of R, giving your answer to 3 decimal places.

(3)

(*c*) Use integration to find the exact value for the area of R.

(5)

21.

4. The rate of decay of the mass of a particular substance is modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{5}{2}x, \quad t \geq 0,$$

where x is the mass of the substance measured in grams and t is the time measured in days.

Given that $x = 60$ when $t = 0$,

(*a*) solve the differential equation, giving x in terms of t. You should show all steps in your working and give your answer in its simplest form.

(4)

(*b*) Find the time taken for the mass of the substance to decay from 60 grams to 20 grams. Give your answer to the nearest minute.

(3)

22.

6. (i) Given that $y > 0$, find

$$\int \frac{3y-4}{y(3y+2)}\,dy.$$

(6)

(ii) (*a*) Use the substitution $x = 4\sin^2\theta$ to show that

$$\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)}\,dx = \lambda \int_0^{\frac{\pi}{3}} \sin^2\theta \, d\theta,$$

where λ is a constant to be determined.

(5)

(*b*) Hence use integration to find

$$\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)}\,dx,$$

giving your answer in the form $a\pi + b$, where a and b are exact constants.

(4)

23.

7. (*a*) Find

$$\int (2x-1)^{\frac{3}{2}}\,dx,$$

giving your answer in its simplest form.

(2)

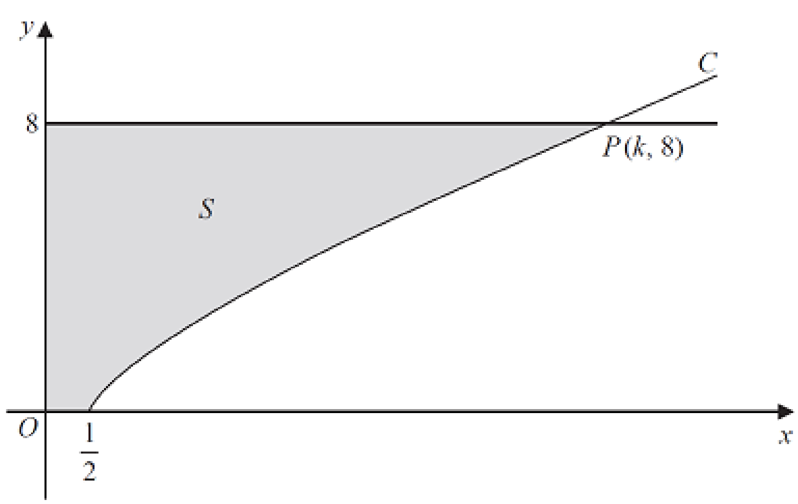

Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = (2x - 1)^{\frac{3}{4}}, \quad x \geq \frac{1}{2}.$$

The curve C cuts the line $y = 8$ at the point P with coordinates $(k, 8)$, where k is a constant.

(*b*) Find the value of k.

(2)

The finite region S, shown shaded in Figure 3, is bounded by the curve C, the x-axis, the y-axis and the line $y = 8$. This region is rotated through 2π radians about the x-axis to form a solid of revolution.

(*c*) Find the exact value of the volume of the solid generated.

(4)

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24.

3.

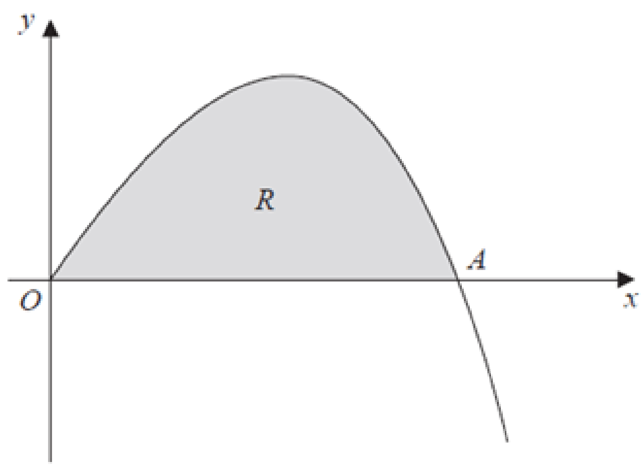

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = 4x - xe^{\frac{1}{2}x}$, $x \geq 0$.

The curve meets the x-axis at the origin O and cuts the x-axis at the point A.

(*a*) Find, in terms of ln 2, the x coordinate of the point A.

(2)

(*b*) Find $\int xe^{\frac{1}{2}x} \, dx$.

(3)

The finite region R, shown shaded in Figure 1, is bounded by the x-axis and the curve with equation $y = 4x - xe^{\frac{1}{2}x}$, $x \geq 0$.

(c) Find, by integration, the exact value for the area of R.
Give your answer in terms of ln 2.

(3)

25.

6.

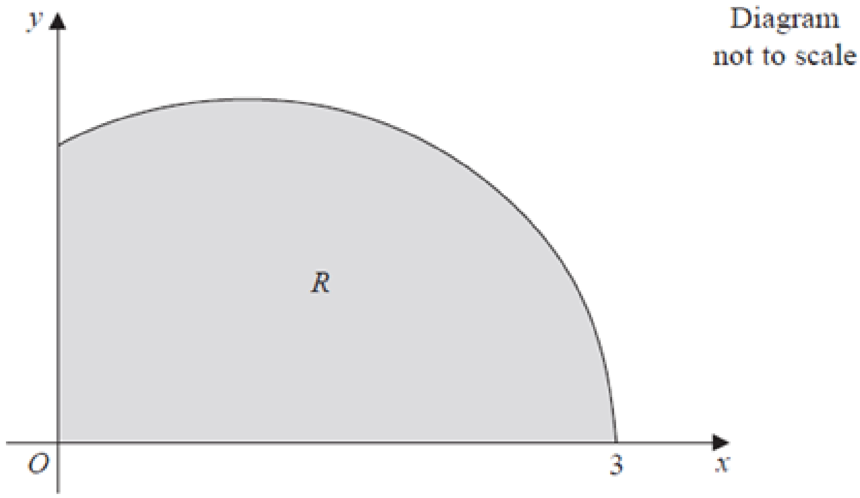

Figure 2

Figure 2 shows a sketch of the curve with equation $y = \sqrt{(3-x)(x+1)}$, $0 \leq x \leq 3$.

The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis, and the y-axis.

(a) Use the substitution $x = 1 + 2\sin\theta$ to show that

$$\int_0^3 \sqrt{(3-x)(x+1)}\ \mathrm{d}x = k\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta\ \mathrm{d}\theta,$$

where k is a constant to be determined.

(5)

(b) Hence find, by integration, the exact area of R.

(3)

26.

7. (*a*) Express $\dfrac{2}{P(P-2)}$ in partial fractions.

(3)

A team of biologists is studying a population of a particular species of animal.

The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{2}P(P-2)\cos 2t, \qquad t \geq 0,$$

where P is the population in thousands, and t is the time measured in years since the start of the study.

Given that $P = 3$ when $t = 0$,

(*b*) solve this differential equation to show that

$$P = \frac{6}{3 - e^{\frac{1}{2}\sin 2t}}$$

(7)

(*c*) find the time taken for the population to reach 4000 for the first time.
Give your answer in years to 3 significant figures.

(3)

27.

8.

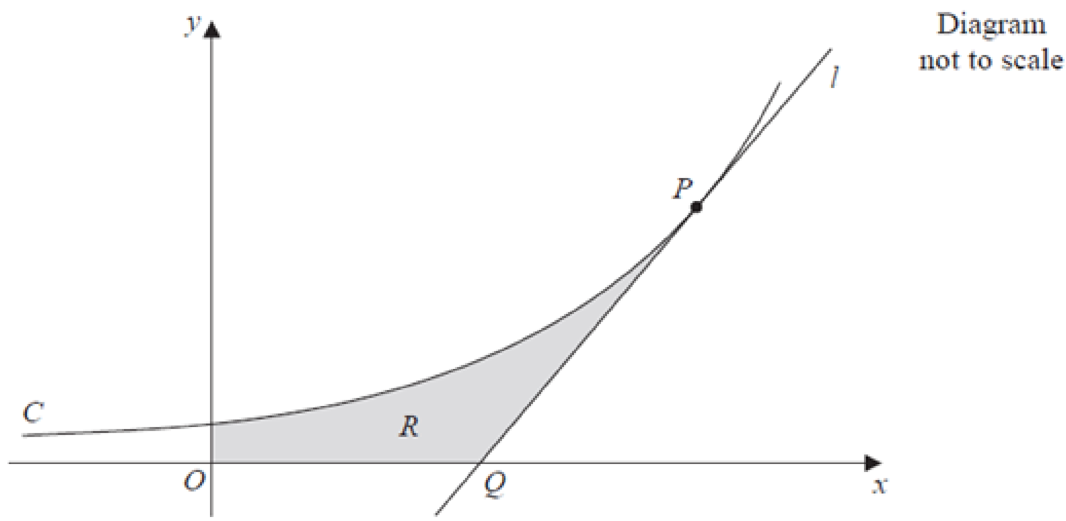

Figure 3

Figure 3 shows a sketch of part of the curve C with equation $y = 3^x$.

The point P lies on C and has coordinates (2, 9).

The line l is a tangent to C at P. The line l cuts the x-axis at the point Q.

(a) Find the exact value of the x coordinate of Q.

(4)

The finite region R, shown shaded in Figure 3, is bounded by the curve C, the x-axis, the y-axis and the line l. This region R is rotated through 360° about the x-axis.

(b) Use integration to find the exact value of the volume of the solid generated.

Give your answer in the form $\frac{p}{q}$, where p and q are exact constants.

[*You may assume the formula* $V = \frac{1}{3}\pi r^2 h$ *for the volume of a cone.*]

(6)

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28.

3.

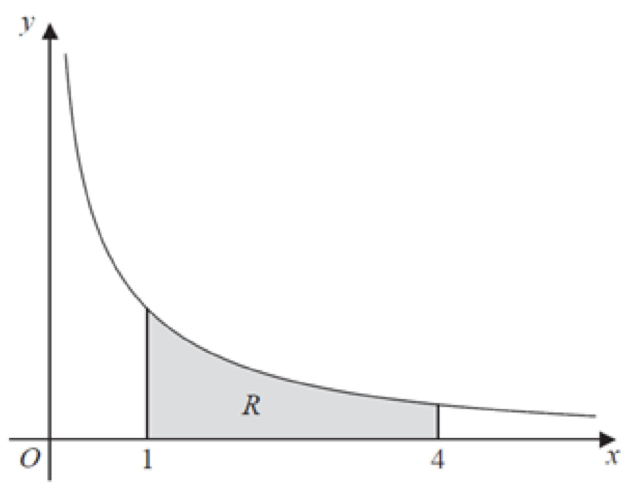

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{10}{2x + 5\sqrt{x}}$, $x > 0$.

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, and the lines with equations $x = 1$ and $x = 4$.

The table below shows corresponding values of x and y for $y = \frac{10}{2x + 5\sqrt{x}}$.

x	1	2	3	4
y	1.42857	0.90326		0.55556

(*a*) Complete the table above by giving the missing value of y to 5 decimal places.
(1)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to find an estimate for the area of R, giving your answer to 4 decimal places.
(3)

(*c*) By reference to the curve in Figure 1, state, giving a reason, whether your estimate in part (*b*) is an overestimate or an underestimate for the area of R.
(1)

(*d*) Use the substitution $u = \sqrt{x}$, or otherwise, to find the exact value of

$$\int_1^4 \frac{10}{2x + 5\sqrt{x}}\,dx$$

(6)

29.

6. (i) Find

$$\int x e^{4x}\,dx$$

(3)

(ii) Find

$$\int \frac{8}{(2x-1)^3}\,dx, \qquad x > \frac{1}{2}$$

(2)

(iii) Given that $y = \frac{\pi}{6}$ at $x = 0$, solve the differential equation

$$\frac{dy}{dx} = e^x \operatorname{cosec} 2y \operatorname{cosec} y$$

(7)

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30.

1. (*a*) Find $\int x^2 e^x \, dx$.

(5)

(*b*) Hence find the exact value of $\int_0^1 x^2 e^x \, dx$.

(2)

31.

3.

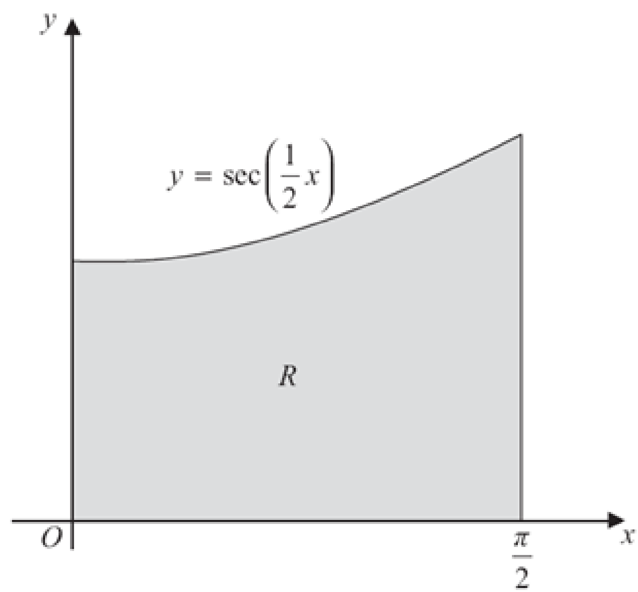

Figure 1

Figure 1 shows the finite region R bounded by the x-axis, the y-axis, the line $x = \frac{\pi}{2}$ and the curve with equation

$$y = \sec\left(\frac{1}{2}x\right), \qquad 0 \leq x \leq \frac{\pi}{2}$$

The table shows corresponding values of x and y for $y = \sec\left(\frac{1}{2}x\right)$.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	1	1.035276		1.414214

(*a*) Complete the table above giving the missing value of y to 6 decimal places.

(1)

(*b*) Using the trapezium rule, with all of the values of y from the completed table, find an approximation for the area of R, giving your answer to 4 decimal places.

(3)

Region R is rotated through 2π radians about the x-axis.

(*c*) Use calculus to find the exact volume of the solid formed.

(4)

32.

5. (*a*) Use the substitution $x = u^2$, $u > 0$, to show that

$$\int \frac{1}{x(2\sqrt{x}-1)}\,\mathrm{d}x = \int \frac{2}{u(2u-1)}\,\mathrm{d}u$$

(3)

(*b*) Hence show that

$$\int_1^9 \frac{1}{x(2\sqrt{x}-1)}\,\mathrm{d}x = 2\ln\left(\frac{a}{b}\right)$$

where a and b are integers to be determined.

(7)

33.

6. Water is being heated in a kettle. At time t seconds, the temperature of the water is θ°C.

The rate of increase of the temperature of the water at any time t is modelled by the differential equation

$$\frac{d\theta}{dt} = \lambda(120 - \theta), \qquad \theta \leq 100$$

where λ is a positive constant.

Given that $\theta = 20$ when $t = 0$,

(a) solve this differential equation to show that

$$\theta = 120 - 100e^{-\lambda t}$$

(8)

When the temperature of the water reaches 100°C, the kettle switches off.

(b) Given that $\lambda = 0.01$, find the time, to the nearest second, when the kettle switches off.

(3)

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34.

2. (a) Use integration to find

$$\int \frac{1}{x^3} \ln x \, dx .$$

(5)

(b) Hence calculate

$$\int_1^2 \frac{1}{x^3} \ln x \, dx .$$

(2)

35.

4.

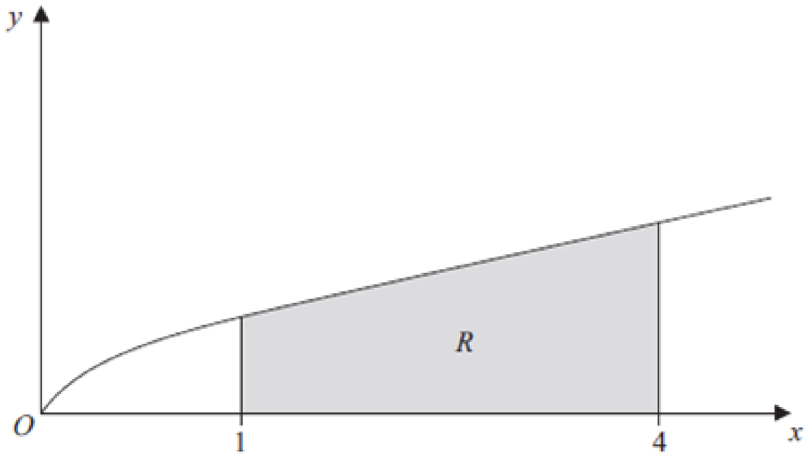

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{x}{1+\sqrt{x}}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, the line with equation $x = 1$ and the line with equation $x = 4$.

(*a*) Copy and complete the table with the value of y corresponding to $x = 3$, giving your answer to 4 decimal places.

(1)

x	1	2	3	4
y	0.5	0.8284		1.3333

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate of the area of the region R, giving your answer to 3 decimal places.

(3)

(*c*) Use the substitution $u = 1 + \sqrt{x}$, to find, by integrating, the exact area of R.

(8)

36.

6.

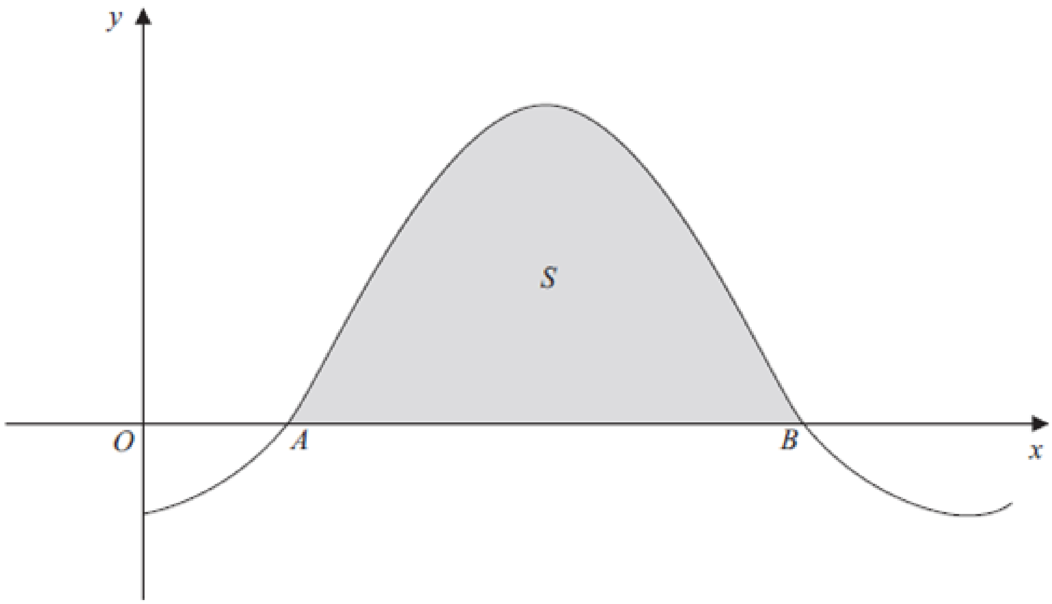

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = 1 - 2\cos x$, where x is measured in radians. The curve crosses the x-axis at the point A and at the point B.

(*a*) Find, in terms of π, the x coordinate of the point A and the x coordinate of the point B.

(3)

The finite region S enclosed by the curve and the x-axis is shown shaded in Figure 3. The region S is rotated through 2π radians about the x-axis.

(*b*) Find, by integration, the exact value of the volume of the solid generated.

(6)

37.

8. A bottle of water is put into a refrigerator. The temperature inside the refrigerator remains constant at 3 °C and t minutes after the bottle is placed in the refrigerator the temperature of the water in the bottle is θ °C.

The rate of change of the temperature of the water in the bottle is modelled by the differential equation

$$\frac{d\theta}{dt} = \frac{(3-\theta)}{125}.$$

(*a*) By solving the differential equation, show that

$$\theta = Ae^{-0.008t} + 3,$$

where A is a constant.

(4)

Given that the temperature of the water in the bottle when it was put in the refrigerator was 16 °C,

(*b*) find the time taken for the temperature of the water in the bottle to fall to 10 °C, giving your answer to the nearest minute.

(5)

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38.

1.

$$f(x) = \frac{1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}.$$

(*a*) Find the values of the constants A, B and C.

(4)

(*b*) (i) Hence find $\int f(x)\ dx$.

(ii) Find $\int_1^2 f(x)\ dx$, leaving your answer in the form $a + \ln b$, where a and b are constants.

(6)

39.

4. Given that $y = 2$ at $x = \frac{\pi}{4}$, solve the differential equation

$$\frac{dy}{dx} = \frac{3}{y\cos^2 x}.$$

(5)

40.

7.

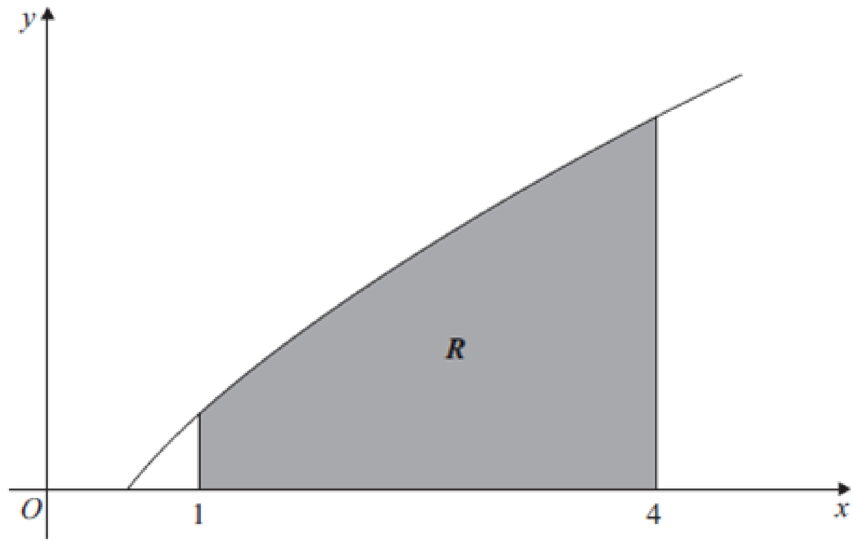

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = x^{\frac{1}{2}} \ln 2x$.

The finite region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 4$.

(*a*) Use the trapezium rule, with 3 strips of equal width, to find an estimate for the area of R, giving your answer to 2 decimal places.

(4)

(*b*) Find $\int x^{\frac{1}{2}} \ln 2x \; dx$.

(4)

(*c*) Hence find the exact area of R, giving your answer in the form $a \ln 2 + b$, where a and b are exact constants.

(3)

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41.

2. (*a*) Use integration by parts to find $\int x \sin 3x \ \mathrm{d}x$.

(3)

(*b*) Using your answer to part (*a*), find $\int x^2 \cos 3x \ \mathrm{d}x$.

(3)

42.

4.

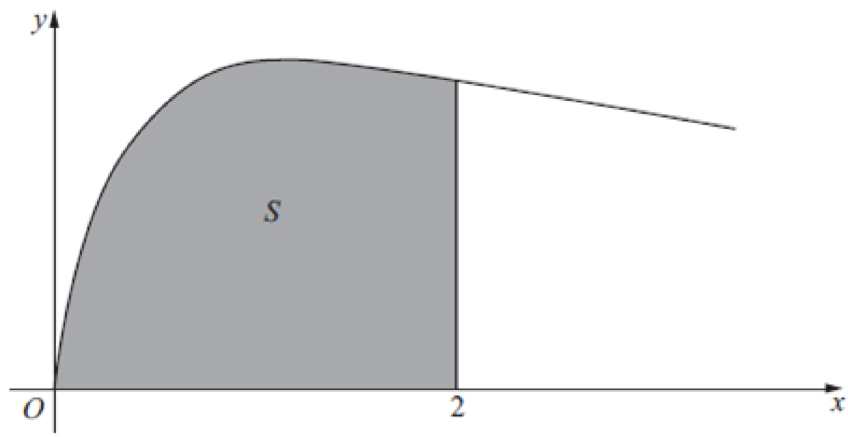

Figure 1

Figure 1 shows the curve with equation

$$y = \sqrt{\left(\frac{2x}{3x^2 + 4}\right)}, \quad x \geq 0.$$

The finite region S, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 2$.

The region S is rotated 360° about the x-axis.

Use integration to find the exact value of the volume of the solid generated, giving your answer in the form $k \ln a$, where k and a are constants.

(5)

43.

6.

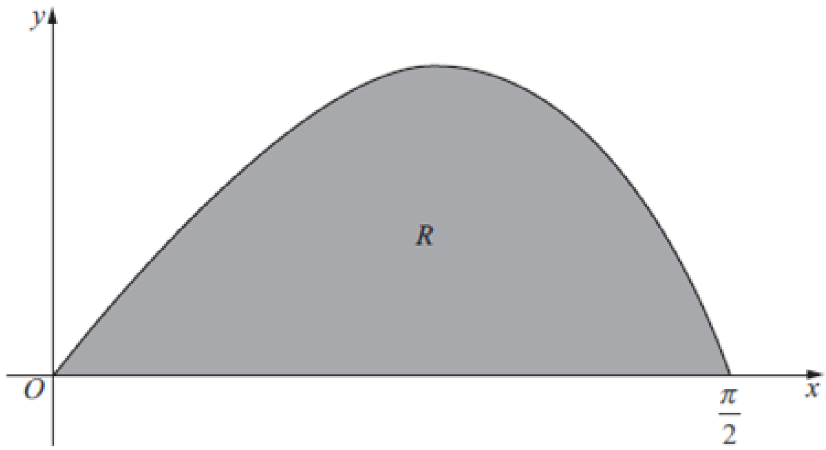

Figure 3

Figure 3 shows a sketch of the curve with equation $y = \dfrac{2\sin 2x}{(1+\cos x)}$, $0 \leq x \leq \dfrac{\pi}{2}$.

The finite region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

The table below shows corresponding values of x and y for $y = \dfrac{2\sin 2x}{(1+\cos x)}$.

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0		1.17157	1.02280	0

(*a*) Complete the table above giving the missing value of y to 5 decimal places.

(1)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 4 decimal places.

(3)

(*c*) Using the substitution $u = 1 + \cos x$, or otherwise, show that

$$\int \frac{2\sin 2x}{(1+\cos x)}\,dx = 4\ln(1+\cos x) - 4\cos x + k,$$

where k is a constant.

(5)

(*d*) Hence calculate the error of the estimate in part (*b*), giving your answer to 2 significant figures.

(3)

44.

8. (*a*) Express $\dfrac{1}{P(5-P)}$ in partial fractions.

(3)

A team of conservationists is studying the population of meerkats on a nature reserve. The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{15}P(5-P), \quad t \geq 0,$$

where P, in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that when $t = 0$, $P = 1$,

(*b*) solve the differential equation, giving your answer in the form,

$$P = \frac{a}{b + ce^{-\frac{1}{3}t}}$$

where a, b and c are integers.

(8)

(*c*) Hence show that the population cannot exceed 5000.

(1)

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45.

4.

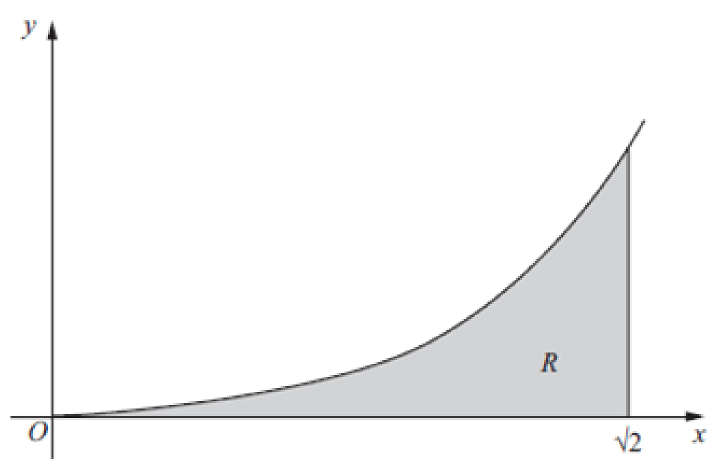

Figure 2

Figure 2 shows a sketch of the curve with equation $y = x^3 \ln(x^2 + 2)$, $x \geq 0$.

The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the line $x = \sqrt{2}$.

The table below shows corresponding values of x and y for $y = x^3 \ln (x^2 + 2)$.

x	0	$\frac{\sqrt{2}}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{3\sqrt{2}}{4}$	$\sqrt{2}$
y	0		0.3240		3.9210

(a) Complete the table above giving the missing values of y to 4 decimal places.
(2)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 2 decimal places.
(3)

(c) Use the substitution $u = x^2 + 2$ to show that the area of R is

$$\frac{1}{2}\int_2^4 (u-2)\ln u \;\, \mathrm{d}u .$$

(4)

(d) Hence, or otherwise, find the exact area of R.
(6)

46.

8. (a) Find $\int (4y+3)^{-\frac{1}{2}} \;\, \mathrm{d}y$.
(2)

(b) Given that $y = 1.5$ at $x = -2$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sqrt{(4y+3)}}{x^2},$$

giving your answer in the form $y = \mathrm{f}(x)$.
(6)

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47.

1. Use integration to find the exact value of $\int_0^{\frac{\pi}{2}} x \sin 2x \ \mathrm{d}x$.

(6)

48.

3. (*a*) Express $\dfrac{5}{(x-1)(3x+2)}$ in partial fractions.

(3)

(*b*) Hence find $\displaystyle\int \frac{5}{(x-1)(3x+2)}\,\mathrm{d}x$, where $x > 1$.

(3)

(*c*) Find the particular solution of the differential equation

$$(x-1)(3x+2)\frac{\mathrm{d}y}{\mathrm{d}x} = 5y, \quad x > 1,$$

for which $y = 8$ at $x = 2$. Give your answer in the form $y = \mathrm{f}(x)$.

(6)

49.

7.
$$I = \int_2^5 \frac{1}{4+\sqrt{(x-1)}}\,\mathrm{d}x.$$

(*a*) Given that $y = \dfrac{1}{4+\sqrt{(x-1)}}$, copy and complete the table below with values of y corresponding to $x = 3$ and $x = 5$. Give your values to 4 decimal places.

x	2	3	4	5
y	0.2		0.1745	

(2)

(*b*) Use the trapezium rule, with all of the values of y in the completed table, to obtain an estimate of I, giving your answer to 3 decimal places.

(4)

(*c*) Using the substitution $x = (u-4)^2 + 1$, or otherwise, and integrating, find the exact value of I.
(8)

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50.

1.

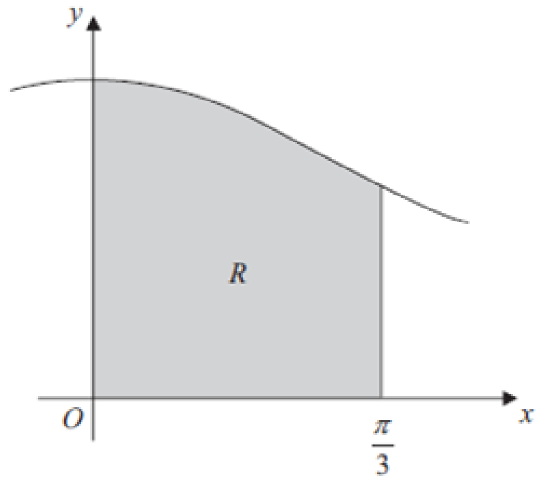

Figure 1

Figure 1 shows part of the curve with equation $y = \sqrt{(0.75 + \cos^2 x)}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the y-axis, the x-axis and the line with equation $x = \frac{\pi}{3}$.

(*a*) Copy and complete the table with values of y corresponding to $x = \frac{\pi}{6}$ and $x = \frac{\pi}{4}$.

x	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
y	1.3229	1.2973			1

(2)

(*b*) Use the trapezium rule

(i) with the values of y at $x = 0$, $x = \frac{\pi}{6}$ and $x = \frac{\pi}{3}$ to find an estimate of the area of R.

Give your answer to 3 decimal places.

(ii) with the values of y at $x = 0$, $x = \frac{\pi}{12}$, $x = \frac{\pi}{6}$, $x = \frac{\pi}{4}$ and $x = \frac{\pi}{3}$ to find a further estimate of the area of R. Give your answer to 3 decimal places.

(6)

51.

2. Using the substitution $u = \cos x + 1$, or otherwise, show that

$$\int_0^{\frac{\pi}{2}} e^{\cos x + 1} \sin x \; dx = e(e - 1).$$

(6)

52.

6. $$f(\theta) = 4\cos^2\theta - 3\sin^2\theta$$

(*a*) Show that $f(\theta) = \frac{1}{2} + \frac{7}{2}\cos 2\theta$.

(3)

(*b*) Hence, using calculus, find the exact value of $\int_0^{\frac{\pi}{2}} \theta\, f(\theta) \; d\theta$.

(7)

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53.

2.

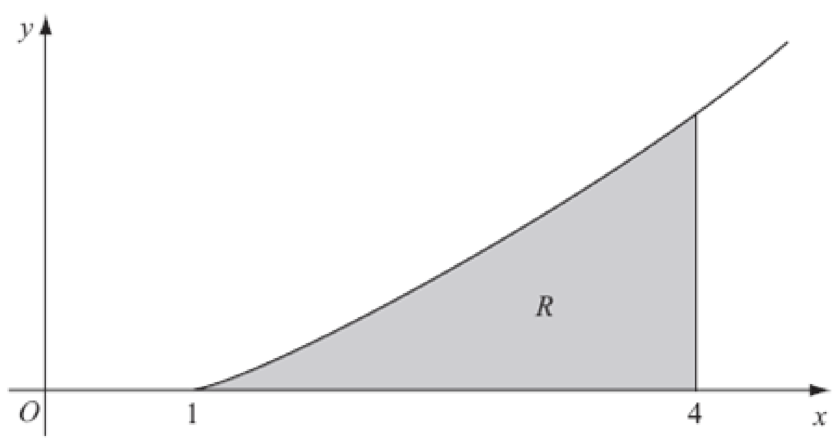

Figure 1

Figure 1 shows a sketch of the curve with equation $y = x \ln x$, $x \geq 1$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 4$.

The table shows corresponding values of x and y for $y = x \ln x$.

x	1	1.5	2	2.5	3	3.5	4
y	0	0.608			3.296	4.385	5.545

(*a*) Copy and complete the table with the values of y corresponding to $x = 2$ and $x = 2.5$, giving your answers to 3 decimal places.

(2)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 2 decimal places.

(4)

(*c*) (i) Use integration by parts to find $\int x \ln x \ \mathrm{d}x$.

(ii) Hence find the exact area of R, giving your answer in the form $\frac{1}{4}(a \ln 2 + b)$, where a and b are integers.

(7)

54.

5. (*a*) Find $\int \frac{9x+6}{x} \, \mathrm{d}x, \quad x > 0$.

(2)

(*b*) Given that $y = 8$ at $x = 1$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(9x+6)y^{\frac{1}{3}}}{x}$$

giving your answer in the form $y^2 = \mathrm{g}(x)$.

(6)

55.

8. (*a*) Using the substitution $x = 2\cos u$, or otherwise, find the exact value of

$$\int_1^{\sqrt{2}} \frac{1}{x^2\sqrt{(4-x^2)}}\,dx.$$

(7)

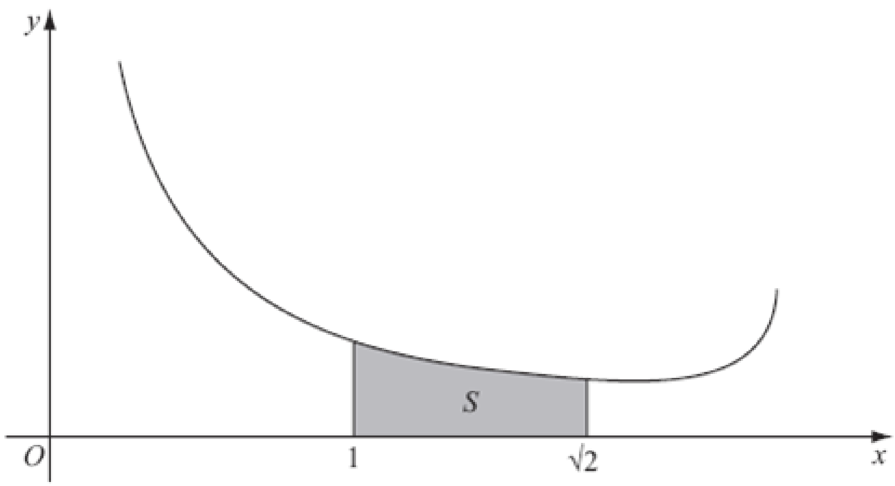

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = \dfrac{4}{x(4-x^2)^{\frac{1}{4}}}$, $0 < x < 2$.

The shaded region S, shown in Figure 3, is bounded by the curve, the x-axis and the lines with equations $x = 1$ and $x = \sqrt{2}$. The shaded region S is rotated through 2π radians about the x-axis to form a solid of revolution.

(*b*) Using your answer to part (*a*), find the exact volume of the solid of revolution formed.

(3)